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Classifying orientifolds by flat n-gerbes

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ABSTRACT: The discrete tensorial charges carried by orientifold planes define n-gerbes in space-time. The simplest way to ensure a consistent string compactification is to require these gerbes to be flat. This results in expressions for the local gerbe-holonomies around each orientifold plane, describing its charges. Inverting the procedure and considering all flat gerbes leads to a classification of orientifold configurations. Requiring that the tadpole is cancelled by adding D-branes, we classify all supersymmetric orientifolds on $T^k/\mathbb{Z}_2$ with 2^k $\mathrm{O}(9-k)$ planes at the fixed points, for $k \leq 6$. For $k = 6$ these theories organize in orbits of the $\mathrm{SL}(2,\mathbb{Z})$ S-duality symmetry of $\mathcal{N} = 4$ supersymmetric gauge theories.

KEYWORDS: Superstrings and Heterotic Strings, String Duality, Superstring Vacua.

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Contents

1. Introduction

An orientifold is a construction in string theory, which involves the gauging of a symmetry of the theory which is a combination of a symmetry of the target space together with the orientation reversal of the string world sheet [26, 27]. If the target space symmetry has a fixed point set, its connected components are called orientifold fixed planes, or just orientifold planes for short (also orientifold lines and points, for sufficiently low dimension of the orientifold plane).

In this article we will restrict slightly further to the best studied category of orientifold planes, the ones that appear in type II string theories and locally look like $\mathbb{R}^{p,1} \times (\mathbb{R}^{9-p})/\mathbb{Z}_2$, where the $\mathbb{Z}_2$ inverts the coordinates on $\mathbb{R}^{9-p}$ (we will review the full action in the next section). These planes preserve 16 of the 32 supersymmetries of the type II theories.

Orientifold planes have various interesting properties, that are intrinsically related. A stack of n D-branes has generically a $\mathrm{U}(n)$ gauge theory propagating on its worldvolume; when the stack of branes is coinciding with an orientifold plane, the group is projected to $\mathrm{O}(n)$ or $\mathrm{Sp}(n/2)$ depending on the kind of orientifold plane. Whether the group is projected to $\mathrm{O}(n)$ or $\mathrm{Sp}(n/2)$ is correlated with a discrete

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charge carried by the orientifold plane. Another charge distinguishes between $O(2n)$ and $O(2n+1)$ symmetry. Although in principle more charges are possible [8], it is not clear what the effect of such charges is on string perturbation theory (if such an effect can be treated in perturbation theory at all), and we will restrict throughout this paper to the two charges mentioned.

The charges are tensorial in nature; they result in fluxes of certain tensorfields. The spectrum of charges can be studied in a suitable formalism such as cohomology or K-theory [2]. In this note we wish to study another aspect of the presence of such charges. In the same sense as that electric and magnetic charges give rise to electromagnetic fields, which can in turn be thought of in terms of a connection on and curvature of bundles, the tensorial charges carried by orientifold planes define tensorfields over space-time. As, in form notation, the one form and higher form tensorfields are very similar, it is tempting to also give a description of higher form tensorfields as connections on "something". A particular formalism that is suited for this is the idea of (abelian) gerbes [9] (see also [20] for a discussion with applications closer to the one in this paper). Gerbes are a generalization of bundles. As a matter of fact, there is a whole tower of mathematical objects, called n-gerbes.

The discrete tensorial charges of the orientifold planes give rise to n-gerbes. For a consistent string background, we have to satisfy the equations of motion. The easiest way to do so is to require the gerbes to be flat, and this is the problem we will study in this paper. A single orientifold plane does not give us any problems, its discrete charges are derived from a cohomology analysis that implicitly tells us that the curvature of the gerbe is at most torsion. For a composition of multiple orientifold planes in a certain geometry this is no longer necessarily true. The analysis in terms of a flux determined by the cohomology of the surrounding space at infinity is no longer valid; the discrete identifications that come with multiple orientifold planes modify the topology of the space at infinity.

In this paper we will study configurations of orientifold planes on the space $\mathbb{R}^{p,1} \times (T^{9-p})/\mathbb{Z}_2$. The orientifold planes at the fixed points are allowed to carry two kinds of discrete charges. We will review some properties of the relevant orientifold planes in section 2. Using some technology developed in a previous paper [3], it is actually not hard to write down the possible flat gerbes. We will do so in section 3.

The constraints we will obtain are not very restrictive for large p, but will become increasingly so for smaller values of p. We conjecture that, at least for the configurations we are studying, there is only one more criterion needed for a consistent compactification. This is the requirement that the RR $(p+1)$ — form tadpole be cancelled. This can always be done by adding a suitable number of Dp- or anti Dp-branes. The most non-suspect backgrounds are the supersymmetric ones, and these require the addition of Dp-branes. We will restrict mostly to these supersymmetric orientifolds, although we will occasionally comment on their non-supersymmetric cousins.

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With the restrictions made, we first study orientifolds with two kinds of planes. and classify these up to geometrical symmetries. This will be done in section 4. The more elaborate case where we no longer restrict to a subset of the possible kinds of planes will be taken up in section 5. The results of these sections, a complete classification for $p \geq 3$, can be found in our tables 1, 2, 3 and 4. Due to a technical problem we cannot proceed beyond 4 noncompact dimensions. In 4 dimensions, the orientifolds we find lead to $\mathcal{N} = 4$ supersymmetric gauge theories. We compare our data with the requirement of the S-duality of these theories in section 6. Finally we summarize and conclude in section 7.

2. Aspects of orientifold planes

The orientifold planes we are interested in locally look like $\mathbb{R}^{p,1} \times (\mathbb{R}^{9-p})/\mathbb{Z}_2$. The generator of the relevant $\mathbb{Z}_2$ transformation is a product of three factors

$$I \cdot \Omega \cdot J \,,$$

where I acts as inversion on the $9-p$ spatial coordinates of $\mathbb{R}^{9-p}$ torus, and Ω is the worldsheet parity operator. Finally, J is either the identity operator for $p = 0, 1 \bmod 4$, or $(-)^{F_L}$ for $p = 2, 3 \bmod 4$, with F_L the left moving fermion number. For p even resp. odd, this $\mathbb{Z}_2$ is a symmetry of IIA resp. IIB theory. As a consequence, in these theories one can find orientifold planes of spatial dimension p, subsequently denoted as Op planes.

For a single Op plane, the space at infinity is $S^{8-p}/\mathbb{Z}_2 = \mathbb{RP}^{8-p}$. Possible tensorial fluxes coming from the orientifold plane are traditionally classified by the cohomologies[1] of $\mathbb{RP}^{8-p}$. We will be interested in orientifold planes with a flux from the NS-NS B-field. The operators I and J acts trivially on this form, but it receives a minus sign from the operator Ω. It is therefore classified by

$$[\mathrm{d}B] = [H] \;\in\; H^3(\mathbb{RP}^{8-p}, \tilde{\mathbb{Z}}) = \mathbb{Z}_2 \,. \tag{2.1}$$

This only makes sense for $p \leq 6$, but for higher values of p one may work with equivariant cohomology [3, 22], which also results in a $\mathbb{Z}_2$ spectrum. The two elements of $\mathbb{Z}_2$ correspond to different signs for a closed string diagram. As is well known, this is correlated with the action of the orientifold projection on the Chan-Paton matrices of the open string. The trivial element of $\mathbb{Z}_2$ results in an $\mathrm{O}(n)$ gauge group[2] on Dp-branes, and the non-trivial one in an $\mathrm{Sp}(n/2)$ gauge group. In the first case, we will denote the plane as Op^-, in the second case as Op^+.

[1] According to current lore, one should use K-theory for the RR fluxes [2]. For the orientifold planes we will consider this gives the same result as cohomology.

[2] Actually, due to subtle effects that are non-perturbative from the viewpoint of the type II description, the actual gauge group is not (necessarily) $\mathrm{O}(n)$. For O9^-, the actual gauge group is $\mathrm{Spin}(32)/\mathbb{Z}_2$ [24]. For lower values of p in Op^- there are in general multiple possibilities, among which $\mathrm{O}(n)$, $\mathrm{SO}(n)$, $\mathrm{Spin}(n)$. The topology of this group does not even have to be such that it

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We will also distinguish an RR charge. Normally the effect of RR charges would be hard to describe in string perturbation theory. The exception to the rule was discovered first for O3 planes [23]. The $\mathrm{SL}(2,\mathbb{Z})$ duality of type IIB theories translates to S-duality of the $\mathcal{N}=4$ supersymmetric Yang-Mills theory that is found on the D3 brane. In the presence of an $\mathrm{O3}^+$ plane, this is an $\mathrm{Sp}(k)$ gauge theory. S-duality maps this to an $\mathrm{SO}(2k+1)$ gauge theory [6], and the NS-NS 2–form charge to an RR 2-form charge. By T-duality, this should extend to other Op planes with $p \neq 3$.

Indeed, one can define a RR $(5-p)$–form charge for Op planes. The relevant cohomologies are given by

$$\begin{aligned} \left[\mathrm{d}C^{5-p}\right] &= \left[G^{6-p}\right] \in H^{6-p}(\mathbb{RP}^{8-p},\mathbb{Z}) = \mathbb{Z}_2, \qquad p \text{ even} \\ \left[\mathrm{d}C^{5-p}\right] &= \left[G^{6-p}\right] \in H^{6-p}(\mathbb{RP}^{8-p},\tilde{\mathbb{Z}}) = \mathbb{Z}_2. \qquad p \text{ odd}\,. \end{aligned} \tag{2.2}$$

This puts a natural upper bound at $p=5$. Above this bound, one still may use T-duality, but the result is that the RR flux is no longer localized at the plane.

There is a simple realization of a plane with such a flux: As $\mathrm{SO}(2k+1)$ is the gauge group that is found on an Op^- plane with an odd number of Dp branes on it, and as even number of branes can always be moved away from the plane, there is obviously a bound state of an Op^- with a single Dp-brane. We will reserve a special notation for this bound state, calling it $\widetilde{\mathrm{O}p}^-$. The tilde is meant to indicate the non-trivial RR flux associated to the plane. The RR $(p+1)$–form charge of this plane is the sum of one unit of Dp brane charge and the charge of the Op^- plane. This shift in $(p+1)$–form charge can also be understood from K-theory [2]. The upper bound of $p=5$ seems to suggest that $\widetilde{\mathrm{O}p}^-$ planes are absent for $p>5$. An attempt to evade this conclusion can be found in [11]. The approach of this paper requires the introduction of a discrete cosmological constant, which would break supersymmetry, and should modify the Einstein equation, excluding flat space (away from the orientifold plane) as a solution. It therefore falls outside the category of planes that we are interested in in this paper.

The final plane is the $\widetilde{\mathrm{O}p}^+$. This carries both the NS-NS 2-form charge and the non-trivial RR $(5-p)$-form charge. It is not possible to form a bound state of an Op^+ with an odd number of Dp-branes. Therefore there is no reason to expect a shift in the RR $(p+1)$-form charge, and also a K-theory computation does not indicate this [2]. It is actually not so easy to distinguish Op^+ and $\widetilde{\mathrm{O}p}^+$. For $p=3$ some differences can be seen in the non-perturbative spectrum of monopoles and dyons in the $\mathcal{N}=4$ theories that arise when a stack of D3 branes coincides with the O3 plane [8]. More on $\widetilde{\mathrm{O}p}^+$ planes can be found in [5, 7, 8, 10, 16, 23].

can be embedded in $\mathrm{Spin}(32)/\mathbb{Z}_2$ [3]. We will simply write down the group that is manifest in perturbation theory, which is $\mathrm{O}(n)$, and ignore symmetry breakings and/or extensions by non-perturbative effects.

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The planes that are relevant to this paper, and their properties are summarized in the following table:

Op plane	group	R_{p+1}	$\mathcal{B}$	$\mathcal{C}$
Op^-	O$(2k)$	-2^{p-4}	0	0
Op^+	Sp(k)	$+2^{p-4}$	1	0
$\widetilde{\mathrm{O}p}^-$	O$(2k+1)$	$1-2^{p-4}$	0	1
$\widetilde{\mathrm{O}p}^+$	Sp(k)	$+2^{p-4}$	1	1

The last three columns denote the charges of a plane: in the third column the RR $(p+1)$-form charge, denoted by R_{p+1}, in the fourth the discrete NS-NS 2-form charge, which we will denote by $\mathcal{B}$ in the remainder of this paper, while the last column denotes the discrete RR $(5-p)$-form charge $\mathcal{C}$.

3. Gerbe holonomies and orientifold charges

For a single Op plane its charges can in principle be measured by surrounding it by a sufficiently large (hyper)surface. For example, it should be possible to measure $\mathcal{B}$ by computing $\int B_2$ over a suitably chosen 2-surface. For a single Op-plane this causes no problems. For a configuration of multiple planes, the various fluxes of the planes have to be patched together. To achieve this, we turn to the formalism of (abelian) gerbes [9]. Some of the material presented here appeared (in a slightly different form, and adapted to a different problem) in [3]. We will repeat some of the discussion because the formalism of gerbes has entered the physics literature rather recently, and to make the paper more self-contained.

Consider an n-gerbe[3] defined on a manifold M, in the pragmatic sense as in [9] (see also [25]). This means that we define a gerbe by transition functions over patches, satisfying suitable cocycle identities. A connection on an n-gerbe is an $(n+1)$-form[4] C_i, defined over the patches U_i of the manifold. On non-empty intersections $U_{ij} = U_i \cap U_j$ the forms over the patches U_i and U_j are related by a gauge transformation $C_i = C_j + \mathrm{d}C_{ij}$. The C_{ij} can be viewed as connections on $(n-1)$ gerbes defined at the overlap patches U_{ij}. To see why, note that as only $\mathrm{d}C_{ij}$ is defined, the C_{ij} are only defined up to closed forms (with integer periods), and hence have their own gauge invariance. On triple intersections $U_i \cap U_j \cap U_k$ there is the consistency condition $d(C_{ij}+C_{jk}+C_{ki}) = 0$ which is compatible with the ambiguity of adding closed forms. Therefore $C_{ij}+C_{jk}+C_{ki}$ and $C_{ij}+C_{jk}+C_{ki}+dC_{ijk}$ define the same cocycle on the triple overlap. The C_{ijk} can be thought of as connections on an $(n-2)$-gerbe, defined

[3] It is customary to call 1-gerbes simply gerbes. 0-gerbes are line bundles [9].

[4] Although some may find the convention to label the gerbe by the number n while it has a $(n+1)$-form connection confusing, it is actually convenient for discussions in physics. It implies that n-gerbes should naturally appear in the description of objects extended in n spatial directions (n-branes), as these couple in a natural way to $(n+1)$-forms [25].

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over the $U_{ijk} = U_i \cap U_j \cap U_k$, which again have their own gauge invariance. This extends all the way until we arrive at zero-forms (functions), which can be thought of as transition functions for a 0-gerbe (that is, a line bundle).

When computing a specific holonomy, it is more convenient to reduce the overlaps to infinitesimal size. Hence we cut up M in pieces M_i by a partition of unity. If the pieces M_i and M_j have a common boundary we denote this M_{ij}; a common boundary between M_i, M_j and M_k is written as M_{ijk} etc. Define $(n+1)$-forms C_i over the pieces M_i; one can think about the M_{ij} as the places where one "jumps" from one patch to another (one can embed the M_i in open patches U_i, then the M_{ij} will be embedded in U_{ij}, etc. which gives our previous description). Again on overlaps, the connections in M_i and M_j are related by the gauge transformations $C_i = C_j + dC_{ij}$. C_{ij} itself is defined over the M_{ij}, and has an intrinsic ambiguity by a closed form. One regards C_{ij} as a connection on an $(n-1)$-gerbe over M_{ij} with a U(1) gauge invariance etc.

The concrete holonomy formula is (note the alternating sign):

$$\int_M C = \sum_i \int_{M_i} C_i - \sum_{ij} \int_{M_{ij}} C_{ij} + \sum_{ijk} \int_{M_{ijk}} C_{ijk} - \cdots . \tag{3.1}$$

This is invariant (for M without boundary) under

$$\begin{aligned} C_i &\to C_i + dL_i \\ C_{ij} &\to C_{ij} + L_i + L_j + dL_{ij} \\ C_{ijk} &\to C_{ijk} + L_{ij} + L_{jk} + L_{ki} + dL_{ijk} \\ &\ \ \ldots \end{aligned} \tag{3.2}$$

Two extreme cases of this formula are for a globally well-defined form C, in which case the sum on the r.h.s. reduces to a single term, and the case when only the last sum of integrals in the expression contributes. These are analogues of the bundle case where physicists are used to either using well-defined connections over large patches, or to "putting the holonomy in the transition functions". For the case of connections on gerbes, there is a much larger freedom to "put" the holonomy somewhere, due to the multiple gauge invariances in (3.2).

Note that, even if the dimension of M is smaller than the degree of the form C, eq. (3.1) gives a perfectly sensible, and not necessarily trivial result. One sets forms of too high degree formally to zero, but this does not necessarily imply that transition gerbes/bundles/functions are trivial. This may seem a rather pathological situation, but will be very useful to us below, because it actually allows us to avoid some technicalities when M has a low dimension.

The above formula can be found in [3], and was more or less inspired by gauge invariance, and inductive reasoning starting with the much simpler case of line bundles

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(0-gerbes). The reader may also want to consult [17] and compare with eq. (8) and figure 5 of that paper for the holonomy formula for 1-gerbes (or just gerbes for short).

In this paper we will use the above formula exclusively on $M = T^k/\mathbb{Z}_2$ (The fact that this $\mathbb{Z}_2$ defines an orientifold instead of an orbifold is relevant for the classification of charges, but not for the gerbe computations that follow). We choose coordinates x_i $(i = 1, \ldots, k)$ in $(\mathbb{R}/2\mathbb{Z})^k$ on T^k, and quotient by the $\mathbb{Z}_2$ reflecting all coordinates. We label the 2^k fixed points of the $\mathbb{Z}_2$ by $x_i = p_i$, $p_i \in (\mathbb{Z}_2)^k$. The periodicity $2\mathbb{Z}$ was chosen to allow us to work with the additive representation of the field of two elements, which we will denote as $\mathbb{Z}_2$. With this convention we can use ordinary addition and multiplication, with reduction modulo 2 understood, in a number of future computations.

The B-field, which is a 2-form, can be regarded as a connection on a (1-)gerbe. We will require this gerbe to be flat, which means that the field strength $H = \mathrm{d}B = 0$. A non-zero H would couple to gravity, and warp the background. We will compute the B-holonomy around an orientifold plane. Because the gerbe is flat, this is independent of the 2-surface we use to compute the holonomy (as long as it has the point p in its interior, and the only possible source for B is at p). For two different surfaces that can be deformed into eachother, the difference in holonomy is expressible as the integral of H over the 3-surface that is swept out in the deformation, which has the two surfaces as boundary, and as H is zero, this difference is 0. With the conventions used, it is possible to write down relatively simple expressions for gerbe holonomy defined over hypersurfaces around the $\mathbb{Z}_2$ fixed points.

We pick a fixed point which we will denote by p, with coordinates p_i. We define a box around this point by using the coordinate ranges $p_{n-2} < x_{n-2} < p_{n-2} + \epsilon, \quad p_i - \epsilon < x_i < p_i + \epsilon$ for $i = (n-1), n$, and localizing the box in the remaining directions by setting $x_i = p_i$ for $i < (n-2)$. The parameter ϵ is not necessarily small; we only require that the box contains at most one possible source, and therefore $0 < \epsilon < 1$. The surface of this box is a 2-surface; we will compute the B-field holonomy over it. This seems to assume that $n \geq 3$, but as noted before, this is not strictly necessary. For $n < 3$, we can still evaluate (3.1), but one truncates the computation by setting forms of too high degree, and the integrals with them formally to 0. We will not attempt to prove in all rigor that this procedure makes sense, but just note that this pragmatic approach reproduces known results.

Let there be a 2-form field

$$\frac{\sum_{i<j<k} b_{ij} \mathrm{d}x_i \mathrm{d}x_j}{4}$$

in the bulk. We can assume that this is a constant 2-form; one can always use gauge transformations to set the 2-form locally to a constant. Another viewpoint is that this 2-form is inherited from T^k, as it is invariant under the $\mathbb{Z}_2$ action that we wish to quotient out, and hence survives the quotient. Because we can take it to be constant on T^4, we can take it constant on the quotient. This fixes the first of the gauge invariances in (3.2), but it is not hard to see that the result of our computation will be gauge independent. We will integrate this over a surface that has the topology of $S^2/\mathbb{Z}_2 = \mathbb{RP}^2$.

From now on we will always assume that the summed over indices are ordered. In the equations below, this will actually present a choice of gauge that is convenient for the choice of surface that we made. The first integral in the holonomy formula is over the faces of the cube; there are 5 of these, of which 4 pair up as opposite faces. The opposite faces do not contribute to the integral, because their contributions cancel (and the form is constant). The only contribution comes from the face at $x_{n-2} = p_{n-2} + \epsilon$. This gives

$$\sum_i \int_{M_i} B_i = b_{n-1,n}\epsilon^2 \,. \tag{3.3}$$

We next consider the transition functions: these are defined at $x_i = p_i$ for $i \leq (n-2)$, $p_{n-1} < x_{n-1} < p_{n-1} - \epsilon$, $p_n - \epsilon < x_n < p_n + \epsilon$. The constant 2-form jumps upon traversing the plane at $x_{n-2} = p_{n-2}$ by an amount $b_{ij}\mathrm{d}x_i\mathrm{d}x_j/2$, and we should write this as the derivative of something. This is inherently ambiguous (by (3.2)), so we fix the gauge and write

$$b_{ij}\mathrm{d}x_i\mathrm{d}x_j = \mathrm{d}\left(\sum_{i<j} b_{ij}x_i\mathrm{d}x_j + b_i\mathrm{d}x_i\right) . \tag{3.4}$$

Here the second term is a closed 1-form that does not contribute to the transition function, but will contribute to the holonomy. Independence of the gerbe from local definitions requires that the b_i are independent of the patch we choose.

We have to integrate this 1-form over the edge of $x_i = p_i$ $(i \leq (n-2))$, $p_{n-1} < x_{n-1} < p_{n-1} + \epsilon$, $p_n - \epsilon < x_n < p_n + \epsilon$. Again only one side contributes (the one with $x_{n-2} = p_{n-2}$, $x_{n-1} = p_{n-1} + \epsilon$), resulting in

$$\left(\sum_i b_{in}p_i + b_n\right)\epsilon + b_{n-1,n}\epsilon^2 \tag{3.5}$$

The last contribution comes from the point at $x_i = p_i$ $(i \leq (n-1))$, $x_n = p_n + \epsilon$. The transition function is

$$\frac{\left(\sum_{i<j} b_{ij}x_ix_j + \sum_i b_ix_i + b\right)}{2} . \tag{3.6}$$

Again the value of b cannot depend on the patch we are in. Inserting values for the coordinates gives

$$\sum_{i<j} b_{ij}p_ip_j + \sum_i b_ip_i + b + \left(\sum_i b_{in}p_i + b_n\right)\epsilon . \tag{3.7}$$

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To find the total holonomy we add the contributions (3.3), (3.5), and (3.7). Because the gerbe is flat, we can actually replace our specific surface by any surface M_p with the point p (and no other source) in its interior. The final result is then:

$$\mathcal{B}(\{p_i\}) \equiv \int_{M_p} B = \sum_{i<j} b_{ij} p_i p_j + \sum_i b_i p_i + b\,. \tag{3.8}$$

This formula is to be computed modulo 2; all coefficients b_{ij}, b_i, b and p_i are elements of $\mathbb{Z}_2$. The actual holonomy is

$$\exp i\pi \mathcal{B}(\{p_i\}) \tag{3.9}$$

and converts the additive representation of $\mathbb{Z}_2$ into its multiplicative one. The holonomy is 1 around an $\mathrm{O}p^-$ or $\widetilde{\mathrm{O}p}^-$, and -1 around a $\mathrm{O}p^+$ or $\widetilde{\mathrm{O}p}^+$. We see that, for sufficiently large k, the holonomy around each of the orientifold planes is completely specified by $k(k-1)/2$ coefficients of the antisymmetric tensor, k coefficients b_i and 1 coefficient b. This is all the freedom one has, and as the NS charges are classified by the equivariant cohomology $H^2_{\mathbb{Z}_2}(T^k, \mathbb{Z}_2)$ [3] one should have

$$H^2_{\mathbb{Z}_2}(T^k/\mathbb{Z}_2) = \mathbb{Z}_2^{\frac{k^2+k+2}{2}} \qquad k \geq 2\,. \tag{3.10}$$

For $k = 1$ we cannot define a 2-form, and one should have $H^2_{\mathbb{Z}_2}(S^1/\mathbb{Z}_2) = \mathbb{Z}_2^2$ because we can only choose b_1 and b. All this is in agreement with computations from [3]. The explicit generators in the appendix D of that paper correspond to a basis for the polynomials (3.8) (which form a vector space over the field $\mathbb{Z}_2$), by simply replacing the generator giving particular values at particular points by the polynomial associating the same values to those points. Finally, for $k = 0$ one considers the equivariant cohomology of $S^0/\mathbb{Z}_2$. As S^0 is two points, $S^0/\mathbb{Z}_2$ is a single point, and one has the trivial result $H^2_{\mathbb{Z}_2}(S^0/\mathbb{Z}_2) = \mathbb{Z}_2$, which is correctly captured by formula (3.8) which in this case consists of the single coefficient b.

We note that for $k \geq 2$ we have $(k^2 + k + 2)/2$ parameters specifying the holonomies around 2^k points. For $k > 2$, the former quantity is smaller, and hence there will be relations among the planes. These can be described in terms of constraints. For $k = 3$ one has 7 parameters describing 8 planes. The single constraint one has is easily derived from

$$\sum_{p_1=0}^{1} \sum_{p_2=0}^{1} \sum_{p_3=0}^{1} \left(\sum_{i<j} b_{ij} p_i p_j + \sum_i b_i p_i + b \right) = 0\,. \tag{3.11}$$

This tells us that summing the $\mathbb{Z}_2$ NS charges of all points gives zero, from which we conclude that the total number of NS charges is even. For $k = 4$ one has 11 parameters for 16 planes, and one has 5 constraints. One of these tells again that the number of planes with NS charge is even. Furthermore, one may also fix a single coordinate and sum over the remaining ones, to see that the number of planes in every

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$T^3/\mathbb{Z}_2$ is even. There are 4 independent choices for the fixed coordinate, and hence in total 5 constraints. In lower dimensions, these constraints become increasingly difficult to analyze, and instead, we will use below a method which stays closer to the original formula's.

Similar formula's can be derived for the RR $(5-p)$-form holonomy around an orientifold plane. In this case however, we have a different result for each dimension. We start with $T^k/\mathbb{Z}_2$ with $k=4$. The fixed points will be O5 planes, which can carry a RR 0-form charge. A zero-form is just a function, flatness requires this to be a constant function so we simply have

$$\mathcal{C}(\{p_i\}) = \int_{M_p} C = c\,. \tag{3.12}$$

For $k=5$ one has a 1-form RR charge. The relevant computation can be found in [3] (where it was computed for the orbifold K3 $T^4/\mathbb{Z}_2$, but the computation here is completely analogous), and one finds

$$\mathcal{C}(\{p_i\}) = \int_{M_p} C = \sum_i c_i p_i + c\,. \tag{3.13}$$

For $k=6$ one has 2-forms, and we can copy our computation for the NS charges.

$$\mathcal{C}(\{p_i\}) = \int_{M_p} C = \sum_{i<j} c_{ij} p_i p_j + \sum_i c_i p_i + c\,. \tag{3.14}$$

For $k=7$ again the relevant formula can be found in [3].

$$\mathcal{C}(\{p_i\}) = \int_{M_p} C = \sum_{i<j<k} c_{ijk} p_i p_j p_k + \sum_{i<j} c_{ij} p_i p_j + \sum_i c_i p_i + c\,. \tag{3.15}$$

The pattern will be clear by now. The holonomy around the orientifold plane is 1 around an Op^- or Op^+, and -1 around a $\widetilde{\mathrm{O}p}^-$ or $\widetilde{\mathrm{O}p}^+$.

Therefore, the pair $(\mathcal{B}(\{p_i\}), \mathcal{C}(\{p_i\}))$ completely determines the identity of the orientifold plane. We define

$$\begin{aligned}
n_- &= \{\#p : \mathcal{B}(\{p_i\}) = 0, \mathcal{C}(\{p_i\}) = 0\} \\
n_+ &= \{\#p : \mathcal{B}(\{p_i\}) = 1, \mathcal{C}(\{p_i\}) = 0\} \\
\tilde{n}_- &= \{\#p : \mathcal{B}(\{p_i\}) = 0, \mathcal{C}(\{p_i\}) = 1\} \\
\tilde{n}_+ &= \{\#p : \mathcal{B}(\{p_i\}) = 1, \mathcal{C}(\{p_i\}) = 1\}\,.
\end{aligned} \tag{3.16}$$

The notation "$\#p$" should be read as "the number of points p with $\ldots$". The numbers $n_-, n_+, \tilde{n}_-$ and $\tilde{n}_+$ count the numbers of Op^-, Op^+, $\widetilde{\mathrm{O}p}^-$ and $\widetilde{\mathrm{O}p}^+$ respectively.

Another crucial number is represented by the RR tadpole. We represent this by the number r, defined as

$$r = 16\,\frac{(n_- + \tilde{n}_-) - (n_+ + \tilde{n}_+)}{(n_- + \tilde{n}_-) + (n_+ + \tilde{n}_+)} - \frac{\tilde{n}_-}{2}\,. \tag{3.17}$$

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This formula expresses tadpole cancellation, where the number of Dp-brane pairs is r. In sufficiently high dimension ($k < 4$) $\tilde{n}_-$ and $\tilde{n}_+$ are actually zero, due to the absence of an RR charge for Op-planes in these dimensions. Of course $n_- + \tilde{n}_- + n_+ + \tilde{n}_+$ is simply the total number of fixed planes 2^k, but we prefer to keep the sum explicit, as it clearly reveals the different ways in which the NS charge and RR charge for orientifold planes result in rank reduction.

For a consistent theory one has to cancel the RR tadpole. If r is positive this can be done by adding r pairs of Dp branes. If r is negative, tadpole cancellation requires the addition of r pairs of anti Dp branes, and this necessarily breaks supersymmetry. Below, we will restrict to r non-negative, and we will refer to configurations with r negative as "breaking supersymmetry". If r is non-negative, one may loosely refer to r as the "rank" of the gauge group, although strictly speaking this is not true, as there are also gauge symmetries coming from Kaluza-Klein reduction on the metric and various forms. These however are not manifest in the orientifold set-up.

By letting the coefficients b, b_i, b_{ij}, and c, c_i, c_{ij}, c_{ijk} taking all possible values, one obtains all flat gerbes over $T^n/\mathbb{Z}_2$. This is too much; many of these gerbes can be related by using coordinate transformations on $T^n/\mathbb{Z}_2$. We will eliminate these redundancies by reducing the polynomials describing the gerbes to suitably chosen standard forms. Also, we will be less interested in the configurations that break supersymmetry (which in the case of a small number of non-compact dimensions outnumber the supersymmetric configurations), and therefore we will require $r \geq 0$.

The formula (3.8) is the same for all values of k. If one has a theory on $T^k/\mathbb{Z}_2$, with a particular configuration of NS charges, one can compactify the theory on (an) additional circle(s). If one does not turn on B-fields over these extra cycles, one can without difficulty T-dualize. The formula for the configuration on $T^{k+n}/\mathbb{Z}_2$ remains the same, which implies that the numbers $(n_- + \tilde{n}_-)$ and $(n_+ + \tilde{n}_+)$ are multiplied by powers of 2.

Upon using a similar procedure on the formula's for the RR charge, (3.12) should be mapped to (3.13) which in turn gets mapped to (3.14), and so on. We know how the top-form (the coefficient in $\mathcal{C}$ with the most indices) transforms; it simply gains the index of the direction in which we dualize. It follows that the same must be true for all the coefficients in the polynomial $\mathcal{C}(\{p_i\})$; all gain an index upon T-dualizing, and the map of the polynomials into each other is now obvious. We also see that the number of planes with RR charge stays the same under T-duality. As a consequence of these rules, the formula (3.17) is invariant under T-duality (as it should be).

All this is perfectly consistent with the T-duality rules ($q = p + 1$):

$$\begin{aligned} \mathrm{O}q^- &\leftrightarrow \mathrm{O}p^- + \mathrm{O}p^- \\ \mathrm{O}q^+ &\leftrightarrow \mathrm{O}p^+ + \mathrm{O}p^+ \\ \widetilde{\mathrm{O}q}^- &\leftrightarrow \mathrm{O}p^- + \widetilde{\mathrm{O}p}^- \\ \widetilde{\mathrm{O}q}^+ &\leftrightarrow \mathrm{O}p^+ + \widetilde{\mathrm{O}p}^+ \,. \end{aligned} \tag{3.18}$$

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We conclude that the formalism we have set up is both self-consistent and consistent with known facts about orientifolds. We now turn to study the holonomy formula's, and derive their implications for the possible configurations of orientifold planes.

4. Models with at most two kinds of planes

Before dealing with the general case, we first study the equations for $\mathcal{B}(\{p_i\})$ and $\mathcal{C}(\{p_i\})$ separately. This amounts to studying configurations where only one of the $\mathbb{Z}_2$ charges, and hence at most two kinds of orientifold planes are present.

4.1 Models with $\mathrm{O}p^-$ and $\mathrm{O}p^+$ only

We will start by first classifying all configurations of NS charges that give rise to flat gerbes. As these are reflected in $\mathcal{B}(\{p_i\})$, we will set $\mathcal{C}(\{p_i\}) = 0$ for the remainder of this subsection. Doing so immediately will give us a list of configurations with $\mathrm{O}p^-$ and $\mathrm{O}p^+$-planes that give rise to flat gerbes. We will still impose the requirement of supersymmetry. Combining $r \geq 0$ with formula (3.17), one sees that in this context this translates into $n_- \geq n_+$. At the end of the subsection we will add some comments on non-supersymmetric configurations.

We are only interested in configurations modulo the action of coordinate transformations on $T^k/\mathbb{Z}_2$. We therefore fix the polynomial $\mathcal{B}(\{p_i\})$ to a particular standard form by using these transformations.

On T^k, the symmetry group would be

$$(\mathbb{R}/2\mathbb{Z})^k \ltimes \mathrm{SL}(k, \mathbb{Z})\,,$$

with $(\mathbb{R}/2\mathbb{Z})^k$ the group of translations $\mathbb{R}^k$ up to periodicity $2\mathbb{Z}^k$, and $Sl(2,\mathbb{Z})$ the mapping class group of the k-torus. The symmetry group of the $\mathbb{Z}_2$ quotient is:

$$\mathbb{Z}_2^k \ltimes \mathrm{SL}(k, \mathbb{Z}_2)\,.$$

The $\mathbb{Z}_2$ quotient breaks the translation group $(\mathbb{R}/2\mathbb{Z})^k$ to $\mathbb{Z}_2^k$ because the periodicity is affected. The group $\mathrm{SL}(k,\mathbb{Z})$ is not broken, but elements of these group that have equal entries modulo 2 have the same action on the $\mathbb{Z}_2$ quotient (as far as the orientifold planes are concerned), and therefore the group is projected to $\mathrm{SL}(k, \mathbb{Z}_2)$.

The elements of the $\mathbb{Z}_2^k$ subgroup are generated by the affine transformations $x_i \to x_i + 1$. The elements of $\mathrm{SL}(k, \mathbb{Z}_2)$ are linear transformations (over the field $\mathbb{Z}_2$ one has $\mathrm{GL}(k, \mathbb{Z}_2) = \mathrm{SL}(k, \mathbb{Z}_2)$). Acting with these on the polynomial $\mathcal{B}(\{p_i\})$, we will now bring it to a standardform.

Pick two coordinates x_i and x_j such that $b_{ij} = 1$. If there are no such coordinates then the quadratic part of the polynomial is in standardform already. If there are,

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then rename $x_i \leftrightarrow x_1$ and $x_j \leftrightarrow x_2$. In the expression for $\mathcal{B}(\{p_i\})$ there now are a number of tems that involve p_1. Write these as

$$p_1 \left(p_2 + \sum_{i \neq 2} b_{1i} p_i + b_1 \right)$$

Setting $x_2 \to \left(x_2 + \sum_{i \neq 2} b_{1i} x_i + b_1 \right)$ eliminates all of these terms except $p_1 p_2$. After this operation the only term in $\mathcal{B}(\{p_i\})$ that depends on p_1 is $p_1 p_2$. One then isolates all terms containing p_2 which can be written as

$$p_2 \left(p_1 + \sum_{i \neq 1} b_{2i} p_i + b_2 \right)$$

and sets $x_1 \to \left(x_1 + \sum_{i \neq 1} b_{2i} x_i + b_2 \right)$. The only term left that still contains p_1 or p_2 is the product $p_1 p_2$. One then repeats the procedure with p_3 and p_4 etc. until done.

We now have transformed to a configuration with only $b_{i,i+1}$ (i odd) possibly non-zero, and if $b_{i,i+1} = 0$, then so are all $b_{j,j+1} = 0$ for $j \geq i$. If m is the smallest odd integer for which $b_{m,m+1} = 0$ then also all $b_i = 0$ for $i < m$. If any of the b_i with $i \geq m$ is non-zero we first interchange (if necessary) x_i with x_m, to set $b_m = 1$. Subsequently one transforms

$$x_m \to \sum_i b_i x_i + b$$

to set all b_i with $i > m$, and b to zero.

With these manipulations we have arrived at what we will use as standardform. The standardform is specified by a number of parameters $b_{i,i+1}$, i odd, $i < m$ for some odd m, a parameter b_m, and a parameter b (which can only be non-zero if $b_m = 0$). These completely specify the NS charges of a given orientifold configuration on $T^k/\mathbb{Z}_2$, up to coordinate transformation (linear or affine).

It is useful to relax for a moment the constraint that b can only be non-zero if $b_m = 0$. Doing this, we note an elegant symmetry: replacing b by $b+1$, one exchanges Op-planes with NS charge, with planes without NS charge. Therefore this symmetry exchanges n_- and n_+. When $b_m = 1$, the transformation $x_m \to x_m + 1$ precisely has the effect $b \to b + 1$. In this case $b \to b + 1$ is an automorphism of the configuration of orientifold planes, and this implies that if $b_m = 1$, $n_+ = n_-$. This in turn implies that $r = 0$ for these models.

On the other hand, if $b_m = 0$, the action of $b \to b + 1$ does not correspond to any coordinate symmetry. In this case $b \to b + 1$ has the effect $r \to -r$. Hence, only one of the configurations can be supersymmetric, and it is not hard to verify that, with the chosen standardform, this is always the one with $b = 0$.

Consequently, the requirement of supersymmetry together with the chosen standardform, leads us to always set $b = 0$.

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$d_{\max}$			n_-	n_+	r
9	$b_{ij} = 0$	$b_i = 0$	2^k	0	16
8	$b_{ij} = 0$	$b_1 = 1$	2^{k-1}	2^{k-1}	0
7	$b_{12} = 1$	$b_i = 0$	$3 \cdot 2^{k-2}$	2^{k-2}	8
6	$b_{12} = 1$	$b_3 = 1$	2^{k-1}	2^{k-1}	0
5	$b_{12} = b_{34} = 1$	$b_i = 0$	$5 \cdot 2^{k-3}$	$3 \cdot 2^{k-3}$	4
4	$b_{12} = b_{34} = 1$	$b_5 = 1$	2^{k-1}	2^{k-1}	0
3	$b_{12} = b_{34} = b_{56} = 1$	$b_i = 0$	$9 \cdot 2^{k-4}$	$7 \cdot 2^{k-4}$	2
2	$b_{12} = b_{34} = b_{56} = 1$	$b_7 = 1$	2^{k-1}	2^{k-1}	0
1	$b_{12} = b_{34} = b_{56} = b_{78} = 1$	$b_i = 0$	$17 \cdot 2^{k-5}$	$15 \cdot 2^{k-5}$	1
0	$b_{12} = b_{34} = b_{56} = b_{78} = 1$	$b_9 = 1$	2^{k-1}	2^{k-1}	0

Table 1: $T^k/\mathbb{Z}_2$ orientifolds with Op^+ and Op^- planes only.

Putting all information together, one arrives at table 1. The first column of this table lists the maximal dimension of non compact space $d_{\max}$. If a certain model exists in $d_{\max}$, one can further compactify it on an additional n-torus, and use T-dualities to deduce the existence of lower dimensional models. The subsequent column lists the non-zero coefficients of the polynomial $\mathcal{B}(\{p_i\})$. This specifies the model completely, and determines the number of various kinds of Op planes, and their location. For easy reference, the numbers n_- and n_+ that follow from $\mathcal{B}(\{p_i\})$ are given in the next columns. These are functions of the parameter k appearing in $T^k/\mathbb{Z}_2$. Of course one should remember that always $k \geq 9 - d_{\max}$. The last column lists the quantity r, which is independent of k, as it should be.

The entries in table 1 exhibits a very simple structure, that is actually easy to understand. First of all, in every dimension exactly one new model appears, and therefore there are $k + 1$ possible models for any value of k. The models in table 1 can be divided in two classes, depending on the value of b_m. For $b_m = 1$, the models have their ranks reduced by powers of 2. The first model in this chain, $b_{ij} = 0$ with rank 16, is just the T-dual of the standard toroidal compactification of type I theory, with its Spin(32)/$\mathbb{Z}_2$ gauge group. The other elements in the chain are T-dual to the various possibilities of compactifying type I theory without vector structure [28, 29, 22, 30]. The values for b_{ij} in the table are also the values for the B-field on the k-torus in the T-dual type I theory.

The class of models with $b_m = 1$ has equal numbers of Op^+ and Op^- planes. The simplest of these is the model with $d_{\max} = 8$. It has one O8^+ and one O8^- plane, and is dual to type IIB on $S^1/\delta\Omega$. This is an orientifold, but here the worldsheet parity Ω is combined with a translation δ over half the circumference of the circle S^1 [31, 22]. Also the other models with $b_m = 1$ (and therefore $r = 0$) can be understood this way. The existence of these models, which have the same number of various orientifold planes, but inequivalent geometries, was first noted in [2, 3]. These appear at even

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values for $d_{\max}$, and can be understood as duals to compactifications of type IIB on $S^1/\delta\Omega \times T^n$, where there are additional B-fields turned on on T^n [16].

Incidentally, we note that, as promised, in low compact dimension ($9 \geq p \geq 6$) our formalism reproduces the known orientifolds [2, 3, 22], even though the original definition of $\mathcal{B}$ as the integral over an $\mathbb{RP}^2$ surrounding the Op plane runs into difficulties here.

Before concluding this section, we briefly return to the class of non-supersymmetric models we discarded, the ones with $b_m = 0, b = 1$. We start with the simplest of these models, described by $b = 1$ and all other coefficients 0. This would be a 10-dimensional theory with Sp(16) group, realized by composing 32 anti-branes with an O9$^+$-plane [21], and its T-duals. Also Sp(16) bundles on a k-torus allow bundles with twisted boundary conditions, and had we carried out our programme while neglecting supersymmetry, we would have found the T-dual descriptions for all of these. The list would simply be the same list as for the type I duals, but with an extra $b = 1$, and the numbers n_- and n_+ interchanged. A last caveat: these models can only be interpreted as representing the flat connections in a classical gauge theory. In absence of supersymmetry, it should be expected that the degeneracy of vacua will be lifted by quantum corrections. The anti Dp-branes break supersymmetry and in particular the total configuration is not a BPS-system. As a consequence, it should be expected that a generic configuration is unstable, and the anti Dp branes are dynamically driven to special locations. This looks like a T-dual description of the localization of the wave function that is well known for finite volume non-supersymmetric gauge theories (see e.g. [1] and references therein). Actually, even this final state may be only an approximate one, as it is not unlikely that such a theory may tunnel by non-perturbative effects into a stabler (supersymmetric?) minimum.

4.2 Models with Op^- and $\widetilde{\mathrm{O}p}^-$ only

We now turn to classification of flat gerbe coming from RR charges only, meaning that in the remainder of this section we will set $\mathcal{B}(\{p_i\})$ to zero. This automatically gives us the classification of orientifolds $T^k/\mathbb{Z}_2$ with Op^- and $\widetilde{\mathrm{O}p}^-$ only. The polynomial $\mathcal{C}(\{p_i\})$ is different for any value of k, and we cannot treat all dimensions simultaneously, like in the previous section.

For $k < 4$ there is no formula for $\mathcal{C}(\{p_i\})$ because in these models the Op-planes cannot be given an RR charge.

In $k = 4$ one has the simple equation (3.12) which says $\mathcal{C}(\{p_i\}) = c$. This equation is invariant under all reparametrizations of $T^k/\mathbb{Z}_2$, and therefore automatically in "standardform". Setting $c = 0$ reproduces the standard compactification, with 16 O5$^-$ planes. Setting $c = 1$ turns all into $\widetilde{\mathrm{O}5}^-$.

In $k = 5$ one has formula (3.13), which is the simple linear equation $\mathcal{C}(\{p_i\}) = \sum_i c_i p_i + c$. We first have to bring this to standardform. If at least one of the c_i is

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$d_{\max}$			n_-	$\tilde{n}_-$	r
5	$c_{[5..k]} = 0$		2^k	0	16
	$c_{[5..k]} = 1$		$2^k - 16$	16	8
4	$c_i = 0$	$c_{[6..k]} = 1$	$2^k - 32$	32	0
3	$c_{12[7..k]} = 1$	$c_{3[7..k]} = 1$	$2^k - 32$	32	0
	$c_{12[7..k]} = c_{34[7..k]} = 1$	$c_{i[7..k]} = 0$	$2^k - 24$	24	4
	$c_{12[7..k]} = c_{34[7..k]} = 1$	$c_{5[7..k]} = 1$	$2^k - 32$	32	0
	$c_{12[7..k]} = c_{34[7..k]} = c_{56[7..k]} = 1$	$c_{i[7..k]} = 0$	$2^k - 28$	28	2

Table 2: $T^k/\mathbb{Z}_2$ orientifolds with $\widetilde{\mathrm{O}p}^-$ and $\mathrm{O}p^-$ planes only.

non-zero, one uses, if necessary $x_1 \leftrightarrow x_i$ to set $c_1 = 1$. Subsequently one transforms $x_1 \to \mathcal{C}(\{x_i\})$ (by which we mean $\mathcal{C}(\{p_i\})$, with p_i replaced by x_i), and ends up with $c_1 = 1$ and all other coefficients zero. Upon using a T-duality in the 1-direction, we see that this is actually T-dual to a configuration that appeared for $k = 4$. The only remaining option is to set all $c_i = 0$, and set $c = 1$. As in $k = 4$, this is invariant under any coordinate transformation. It results in a configuration with 32 $\widetilde{\mathrm{O}4}^-$ planes.

The one-form in $k = 5$ can be interpreted as a connection on a bundle. In this case we have IIA theory and of course in this case the total space of the bundle is interpreted as the manifold upon which M-theory is compactified. Orientifold planes with RR 1-form charge correspond to twists of this bundle [5, 10], and hence the classification of bundles is the same as the classification of those configurations that can be lifted to M-theory (see the remarks in [3, section 3.2.6]).

In $k = 6$ we have equation (3.14). This is the same formula as for the B-fields, and one can use the tricks of the previous section to rewrite any configuration into a standard form. Again $c = 0$, because of the tadpole requirement. The configurations $c_1 = 1$, and the configuration $c_{12} = 1$ are related by T-dualities to previously considered models. The other ones are new ones.

For $k > 6$ our previous methods fail. Essentially this is because we do not know how to rewrite the tri-linear term in (3.15) or higher order terms appearing for still higher values of k to a standard form. Because of this technical problem our classification of configurations with RR charges will not proceed beyond $k = 6$.

Our results of this section are compiled in table 2. The table has the same structure as table 1 except that now $\mathrm{O}p^+$ planes, and therefore n_+ are absent. Instead $\widetilde{\mathrm{O}p}^-$ planes appear, with their associated quantity $\tilde{n}_-$. The only other new element in table 2 is the notation we introduced in the coefficients of $\mathcal{C}(\{p_i\})$. As explained previously, these coefficients gain indices when T-dualizing. We tried to capture this in the notation $[m..k]$. The idea is that this represents a string of subsequent indices ranging from m to k. If $k < m$ it represents an empty string. As an example, the notation $c_{12[7..k]}$ stands for c_{12} if $k = 6$, for c_{127} if $k = 7$, for c_{1278} if $k = 8$, etcetera.

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The entries in this table have a much more irregular structure than the previous table. The first non-trivial theory in table 2 (the second entry) is T-dual to a type I compactification on T^4 with a so-called non-trivial quadruple of holonomies [3, 12, 15]. These belong to a class of only fairly recently discovered compactifications of non-abelian gauge theories on tori, having flat bundles that are not connected via flat connections to the trivial flat connection, but are also not characterized by their topology.

The entry under $d_{\max} = 4$ is another example of such a compactification, this time T-dual to a type I compactification on T^5 with a non-trivial quintuple of holonomies [3, 12]. The interpretation of the theories listed under $d_{\max} = 3$ is less straightforward, although their holonomies are fairly easy to write down. As explained in [15, section 5], they can be obtained by "superposing" various quadruple configurations, and reducing Dp branes modulo 2.

Besides flat gauge bundles with quadruples and quintuples, there also exist flat gauge bundles with triples. These last bundles were thoroughly studied in a number of papers [4, 12, 13]. Unfortunately, there are severe constraints on the existence of such compactifications in string theory, and as a matter of fact non-trivial triples do not give rise to flat gauge bundles in $\mathrm{Spin}(32)/\mathbb{Z}_2$ string theory [3]. Although non-trivial flat bundles on higher dimensional tori can be realized, the systematics for these theories is not completely understood yet from the gauge theory side.

We conclude this section with a few remarks on the non-supersymmetric models that were discarded. With the interpretation of an $\widetilde{\mathrm{O}p}^-$ plane as a bound state of an Op^- plane with a Dp-brane, and the simple formula for r for the theories in this section

$$r = 16 - \frac{\tilde{n}_-}{2} \tag{4.1}$$

one may interpret these as theories that fail to be supersymmetric because of an insufficient number of Dp branes in the theory. Indeed, were it not for tadpole cancellation, these theories would make perfect sense as dual descriptions to compactifications of $\mathrm{O}(n > 32)$ open string theories. With the requirement of tadpole cancellation, one is automatically forced to introduce $-r$ pairs of anti Dp branes. A pair of anti Dp branes will feel an attractive force towards an $\widetilde{\mathrm{O}p}^-$ plane, and having arrived at this plane should annihilate with the Dp brane there to form a bound state of an Op^- plane with an anti Dp brane. Hence these theories will finally have $(\tilde{n}_- + r)$ $\widetilde{\mathrm{O}p}^-$ planes (remember that r is negative for non-supersymmetric theories) and $-r$ (non-supersymmetric) bound states of Op^- with a single anti Dp on top.

The positions of these planes are not fixed; by Dp brane-antibrane pair creation in the bulk and letting such pairs annihilate with Dp or anti Dp's at the orientifold planes one can permute $\widetilde{\mathrm{O}p}^-$ planes with a bound state of Op^- with anti Dp. Note however that these processes can never convert an Op^- into an $\widetilde{\mathrm{O}p}^-$ or the bound state of Op^- and anti Dp. Again, in the absence of supersymmetry, the degeneracy in

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$d_{\max}$		n_-	n_+	$\tilde{n}_-$	$\tilde{n}_+$	r
5	$b_{12}=1$ $c_{[5..k]}=1$	$3\cdot 2^{k-2}-12$	$2^{k-2}-4$	12	4	2
4	$b_1=1$ $c_{1[6..k]}=1$	2^{k-1}	$2^{k-1}-16$	0	16	0_a
	$b_{12}=1$ $c_{1[6..k]}=1$	$3\cdot 2^{k-2}-8$	$2^{k-2}-8$	8	8	4
	$b_{12}=1$ $c_{1[6..k]}=c_{[6..k]}=1$	$3\cdot 2^{k-2}-16$	2^{k-2}	16	0	0_b
	$b_{12}=b_{34}=1$ $c_{1[6..k]}=1$	$5\cdot 2^{k-3}-8$	$3\cdot 2^{k-3}-8$	8	8	0_c

Table 3: $T^k/\mathbb{Z}_2$ orientifolds with all kinds of planes, $d_{\max}>3$

vacuum configurations is not protected; the actual vacuum should be a superposition of all possible semiclassical vacua. Again we are assuming here that this vacuum is "sufficiently" stable, it is conceivable that by non-perturbative effects this theory may tunnel to another vacuum which is not at all resembling this orientifold description. However, under suitable circumstances the time-scale for such a process may be much longer than the time scale that the orientifold would need to find its metastable vacuum.

5. Mixed cases: All kinds of planes

Finally we will tackle the general case, where both $\mathcal{B}(\{p_i\})$ and $\mathcal{C}(\{p_i\})$ can be non-zero polynomials. We will omit the cases $\mathcal{B}(\{p_i\})=0$ and $\mathcal{C}(\{p_i\})=0$ as their descriptions can be found in the previous sections. Because of the problems with $\mathcal{C}(\{p_i\})$ for $k>6$, our classification will terminate at $k=6$. In $k=6$ the computations are very lengthy, although relatively straightforward. For the ease of the reader we will first present and discuss our results, and present the justifying computations later.

Our results can be found in the tables 3 and 4. These tables have the same structure as the tables 1 and 2, except that now one has 4 columns to count all the kinds of orientifold planes. We will explain in section 6 why we have attached an index to some of the zero's in the last column.

The theories described in the tables 3 and 4 are "mixed" in multiple ways. First of all, "mixed" refers to the presence of both NS and RR charges in the characterization of the orientifold planes. But they are also mixed in the sense that their dual description involves a mixture of the elements of the dual descriptions in the previous sections.

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$d_{\max}$		n_-	n_+	$\tilde{n}_-$	$\tilde{n}_+$	r
3	$b_1 = 1$ $c_{1[7..k]} = 1$	2^{k-1}	$2^{k-1} - 32$	0	32	0
	$b_{12} = 1$ $c_{12[7..k]} = c_{34[7..k]} = 1$	$3 \cdot 2^{k-2} - 12$	$2^{k-2} - 12$	12	12	2
	$b_{12} = 1$ $c_{1[7..k]} = 1$	$3 \cdot 2^{k-2} - 16$	$2^{k-2} - 16$	16	16	0_a
	$b_{12} = 1$ $c_{12[7..k]} = 1$	$3 \cdot 2^{k-2}$	$2^{k-2} - 16$	0	16	8
	$b_{12} = 1$ $c_{12[7..k]} = c_{1[7..k]} = 1$	$3 \cdot 2^{k-2} - 16$	2^{k-2}	16	0	0_a
	$b_{12} = 1$ $c_{15[7..k]} = c_{34[7..k]} = 1$	$3 \cdot 2^{k-2} - 16$	$2^{k-2} - 8$	16	8	0_c
	$b_{12} = 1$ $c_{13[7..k]} = c_{2[7..k]} = c_{3[7..k]} = 1$	$3 \cdot 2^{k-2} - 16$	$2^{k-2} - 16$	16	16	0_b
	$b_{12} = 1$ $c_{14[7..k]} = c_{23[7..k]} = 1$	$3 \cdot 2^{k-2} - 16$	$2^{k-2} - 8$	16	8	0_d
	$b_{12} = b_3 = 1$ $c_{12[7..k]} = c_{23[7..k]} = 1$	2^{k-1}	$2^{k-1} - 16$	0	16	0_b
	$b_{12} = b_3 = 1$ $c_{12[7..k]} = c_{3[7..k]} = 1$	2^{k-1}	$2^{k-1} - 32$	0	32	0
	$b_{12} = b_{34} = 1$ $c_{12[7..k]} = 1$	$5 \cdot 2^{k-3} - 4$	$3 \cdot 2^{k-3} - 12$	4	12	2
	$b_{12} = b_{34} = 1$ $c_{13[7..k]} = 1$	$5 \cdot 2^{k-3} - 8$	$3 \cdot 2^{k-3} - 8$	8	8	0_d
	$b_{12} = b_{34} = 1$ $c_{13[7..k]} = c_{24[7..k]} = 1$	$5 \cdot 2^{k-3} - 8$	$3 \cdot 2^{k-3} - 16$	8	16	0_d
	$b_{12} = b_{34} = 1$ $c_{12[7..k]} = c_{34[7..k]} = 1$	$5 \cdot 2^{k-3}$	$3 \cdot 2^{k-3} - 24$	0	24	4
	$b_{12} = b_{34} = 1$ $c_{15[7..k]} = c_{34[7..k]} = 1$	$5 \cdot 2^{k-3} - 8$	$3 \cdot 2^{k-3} - 8$	8	8	0_c
	$b_{12} = b_{34} = b_5 = 1$ $c_{12[7..k]} = c_{34[7..k]} = c_{5[7..k]} = 1$	2^{k-1}	$2^{k-1} - 32$	0	32	0
	$b_{12} = b_{34} = b_{56} = 1$ $c_{12[7..k]} = c_{34[7..k]} = c_{56[7..k]} = 1$	$9 \cdot 2^{k-4}$	$7 \cdot 2^{k-4} - 28$	0	28	2

Table 4: $T^k/\mathbb{Z}_2$ orientifolds with all kinds of planes, $d_{\max} = 3$

First consider the models where less than half of the Op planes carries an NS charge. As explained in section 4.1 such models are T-dual to type I theories with-

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out vector structure. In section 4.2 we explained that for topologically trivial, flat $\mathrm{Spin}(32)/\mathbb{Z}_2$ bundles there are components in the gauge theory moduli space that are not connected to the trivial flat bundle via flat connections, and that these give rise to new orientifold models. It should come as no surprise that also for topologically non-trivial bundles there are many different components in the moduli space of flat bundles [4], and that these give rise to different orientifold descriptions [3, 14].

The models where half of the Op-planes carry NS charges can still be identified with duals to the type IIB-theory on $S^1/\delta\Omega$. Because in this case $n_- + \tilde{n}_- = n_+ + \tilde{n}_+$, a glance at formula (3.17) will teach us supersymmetry requires that $\tilde{n}_- = 0$. This is also intuitively clear; as the tadpole is already cancelled because of the balance between planes with and without NS charge, there can be no excess Dp-branes forming bound states with Op^- planes.

This argument does not rule out the presence of $\widetilde{\mathrm{O}p}^+$ planes in these theories, and indeed various solutions with these planes exist. It is however not easy to describe accurately the distinction between these theories, and the ones where one replaces the $\widetilde{\mathrm{O}p}^+$ planes by Op^+-planes. Both are duals of the type IIB-theory on $S^1/\delta\Omega$, and their difference is due to rather subtle RR phases on the torus upon which the theory is compactified, which are rather poorly understood at present.

Also for the (large number of) non-supersymmetric theories that are discarded there, it is hard to make general statements. In principle the starting point is again clear: when one computes r to be negative, one adds anti Dp branes to cancel the tadpole. Anti Dp branes can annihilate with Dp branes at $\widetilde{\mathrm{O}p}^-$ planes, and the endpoint of the whole process is depending on r and $\tilde{n}_-$. If $-r > \tilde{n}_-$ there will be free anti Dp branes left. These will then probably be driven to some (meta)stable non-supersymmetric configuration of minimal vacuum energy. If $-r < \tilde{n}_-$ all Dp branes at $\widetilde{\mathrm{O}p}^-$ planes will be annihilated, and one has a configuration where all Dp and anti Dp branes form bound states with Op^- planes, and the true vacuum will be a superposition of all possible configurations. Finally if $-r = \tilde{n}_-$, all anti Dp branes will annihilate, and the final state is, at least semiclassically, unique.[5]

We will now turn to the actual computations that lead to the tables 3 and 4. We start with the easiest computation, for $k = 4$. If at any place the reader gets weary of the computations, he is encouraged to skip the remainder of this section and turn to section 6.

5.1 5+1 dimensional theories

Our general line of attack in this subsection and the following ones will be as follows. First we fix a configuration of NS charges. This amounts to picking an entry from table 1, and inserting the coefficients in the polynomial $\mathcal{B}(\{p_i\})$. After having fixed

[5] An example of such an orientifold is $T^6/\mathbb{Z}_2$, with $\mathcal{B} = p_1p_2+1$, and $\mathcal{C} = p_1p_2$. This demonstrates that this set of models is not empty.

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this we consider in all generality all possibilities for $\mathcal{C}(\{p_i\})$. We have to do so, because the fact that we have already used part of the symmetries to fix $\mathcal{B}(\{p_i\})$ means that we have less freedom to manipulate $\mathcal{C}(\{p_i\})$.

It will be clear that there are two factors determining the difficulty of this process: 1) When we will go to lower dimensions (higher k) the polynomial $\mathcal{C}(\{p_i\})$ will become a more complicated expression; and 2) the more coefficients of $\mathcal{B}(\{p_i\})$ are fixed, the fewer symmetries we have at our disposal. This is why the first computations we will present are almost trivial, while by the end of the third subsection they will have grown very tedious. In these subsections, when we have found a supersymmetric model we will compute the row of 5 numbers $(n_-, n_+, \tilde{n}_-, \tilde{n}_+, r)$ and list it.

Let us apply the procedure to the case of $k = 4$. We fix $\mathcal{B}$, by picking one of the first 4 non-trivial entries in table 1 (as we want something that is not in our tables yet, we require $\mathcal{B}(\{p_i\}) \neq 0$). For $p = 5$ the expression for $\mathcal{C}(\{p_i\})$ is simple: equation (3.12) has only one coefficient. To look for new configurations we have to set $\mathcal{C}(\{p_i\}) = 1$.

Then we have the following cases:

- $b_1 = 1$: Setting $c = 1$ breaks supersymmetry.
- $b_{12} = 1$: Setting $c = 1$ gives the model (0,0,12,4,2).
- $b_{12} = b_3 = 1$: $c = 1$ breaks supersymmetry.
- $b_{12} = b_{34} = 1$: $c = 1$ breaks supersymmetry.

Hence in 6 dimensions, after a few straightforward computations we found one new model. This model made a brief appearance in [3], and was more thoroughly described in [16]. In the next subsection we will find models that have not appeared in the literature thus far.

5.2 4+1 dimensional theories

Again we start by fixing $\mathcal{B}(\{p_i\})$, by picking an entry from table 1. The expression for $\mathcal{C}(\{p_i\})$ is still relatively simple (see eq. (3.13)).

We will use a number of symmetries and properties of the expressions for $\mathcal{B}(\{p_i\})$ to fix the polynomial $\mathcal{C}(\{p_i\})$ to a standardform without affecting $\mathcal{B}(\{p_i\})$. These coordinate transformations play an even more prominent role in fixing a standard form for the 4-dimensional theories.

A very useful and important property is, that the set of indices on the non-zero b_{ij}, b_i has an upper bound. For $k = 5$ this leads immediately to the following simplification. Let m be the smallest integer that does not occur in the indices appearing on the non-zero entries of b_{ij}, b_i. If any of the c_i with $i \geq m$ is non-zero, one first relabels coordinates (only coordinates with $i \geq m$ should be relabelled !) to set $c_m = 1$. Subsequently one can transform away all c_i with $i \neq m$, as well as c, by

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setting $x_m = \mathcal{C}(\{x_i\})$, while leaving all other coordinates invariant. Because c_m was non-zero, this transformation is non-singular, and because the coordinates x_i with $i \neq m$ are not affected, this does not change the expression for $\mathcal{B}(\{p_i\})$. We hence end up with a configuration given by $\mathcal{B}(\{p_i\})$ and $\mathcal{C}(\{p_i\}) = p_m$. Now applying a T-duality in the m-direction, we see that this configuration is related to one that already appeared one dimension higher. Summarizing: If any of the c_i with $i \geq m$ is non-zero, the configuration is related to one considered before and therefore it is either ruled out, or already in our table. Therefore we can restrict to $c_i = 0$ for $i \geq m$.

We now turn to the c_i, with $i < m$. If $b_{i,i+1} = 1$ for some (odd) i, then one can assume that $c_{i+1} = 0$, because if it is not, we can set it to zero in the following way: If $c_i = 0$, $x_i \leftrightarrow x_{i+1}$ (other coordinates invariant) leaves $\mathcal{B}(\{p_i\})$ invariant, but sets c_{i+1} to zero; if $c_i = 1$, setting $x_i \to x_i + x_{i+1} + 1$ leaves $\mathcal{B}(\{p_i\})$ invariant but removes p_{i+1} dependence of $\mathcal{C}(\{p_i\})$.

The previous trick will already eliminate many c_i's, but we can still do more if we have $b_{12} = b_{34} = 1$. In the way just described one sets c_2 and c_4 to zero, but one can also eliminate a non-zero c_3. Either $c_1 = 0$, and one uses $(x_1, x_2) \leftrightarrow (x_3, x_4)$; or $c_1 = 1$ in case one uses $x_1 \to x_1 + x_3$, $x_4 \to x_2 + x_4$. In either case $\mathcal{B}(\{p_i\})$ remains invariant, but c_3 is eliminated. Hence in this case we only have to consider non-zero c_1 and possibly c_5.

The following argument also presents a simplification in 5-dimension, but will in particular be a crucial shortcut to some computations in 4-dimensions. As remarked before, in models where the number of planes with NS charge is equal to the number of planes without NS charge, it is impossible to have $\widetilde{\mathrm{O}p}^-$ planes while preserving supersymmetry. So, for these theories, instead of computing r case by case, we can also simply demand absence of $\widetilde{\mathrm{O}p}^-$ planes in these models. This amounts to requiring that, whenever $\mathcal{B}(\{p_i\})$ gives the value 0 at a certain plane, we should also require $\mathcal{C}(\{p_i\})$ to give 0 at that particular plane. Solving for this constraint fixes most, sometimes even all coefficients in the polynomial $\mathcal{C}(\{p_i\})$.

After these preliminary considerations we simply check the remaining options case by case:

- $b_1 = 1$: As explained, we set $c_i = 0$ for $i > 1$. Supersymmetry requires absence of $\widetilde{\mathrm{O}4}^-$ planes. Then one quickly deduces that $c = 0$ (because that would turn planes at $p_i = 0$ into $\widetilde{\mathrm{O}4}^-$). Hence the only non-trivial possibility is $c_1 = 1$, which gives $(16, 0, 0, 16, 0)$.

- $b_{12} = 1$: We can set $c_i = 0$ for $i > 1$. then the only options left are:

  1. $c = 1$: Breaks supersymmetry.
  2. $c_1 = 1$: Leads to the model (16,0,8,8,4).

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3. $c_1 = c = 1$: Leads to (8,8,16,0,0).

- $b_{12} = b_3 = 1$: Set $c_i = 0$ for $i > 3$ and $c_2 = 0$. Supersymmetry requires absence of $\widetilde{\mathrm{O4}}^-$ planes. Then one quickly deduces that $c = 0$ (because that would turn planes at $p_1 = p_3 = 0$ into $\widetilde{\mathrm{O4}}^-$) that $c_1 = 0$ (with planes at $p_1 = 1$, $p_3 = 0$), and $c_3 = 0$ (with planes at $p_1 = p_3 = 1$). Hence supersymmetry requires $\mathcal{C}(\{p_i\}) \equiv 0$, and we find no new models.

- $b_{12} = b_{34} = 1$: Set $c_i = 0$ for $i > 1$.

  1. $c = 1$: Breaks supersymmetry.
  2. $c_1 = 1$: Leads to (12,4,8,8,0).
  3. $c_1 = c = 1$: Breaks supersymmetry.

- $b_{12} = b_{34} = b_5 = 1$: Set $c_2 = c_3 = c_4 = 0$. Supersymmetry requires absence of $\widetilde{\mathrm{O4}}^-$ planes. Then one quickly deduces that $c = 0$ (because that would turn the plane at $p_i = 0$ into $\widetilde{\mathrm{O4}}^-$) and that $c_1 = 0$ (with planes at $p_1 = 1$), and $c_5 = 0$ (with planes at $p_1 = p_5 = 1$). Hence supersymmetry requires $\mathcal{C}(\{p_i\}) \equiv 0$ and we find no new models.

With a little more effort than in the previous subsection, we have identified in total 4 new models.

5.3 3+1-dimensional theories

In this dimension computations get very tedious. This is to some extent clear from the expression for $\mathcal{B}(\{p_i\})$ (3.8) and $\mathcal{C}(\{p_i\})$ (3.14). In a very literal sense, they are both equally complicated, but after having fixed a standardform of $\mathcal{B}(\{p_i\})$ there are only relatively few possibilities left to manipulate $\mathcal{C}(\{p_i\})$.

One category of theories is still (relatively) simple to tackle: the ones with equal number of planes with NS charge and planes without NS charge. Supersymmetry imposes the absence of $\widetilde{Op}^-$ planes, and having realized that, the computation to be done is straightforward:

- $b_1 = 1$.

  The absence of $\widetilde{\mathrm{O3}}^-$ planes restricts the polynomial $\mathcal{C}(\{p_i\})$ to

$$\mathcal{C}(\{p_i\}) = p_1 \left(\sum_{j \neq 1} c_{1j} p_j + c_1 \right) . \tag{5.1}$$

  If at least one of the c_{1j} is non-zero, we use (if necessary) $x_2 \leftrightarrow x_j$ to set $c_{12} = 1$. Subsequently we transform $x_2 \to (\sum_{j \neq 1} c_{1j} x_j + c_1)$. Hence we are left with the following inequivalent possibilities:

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1. $c_{12} = 1$: One can use T-duality in 2-direction to relate this to $b_1 = c_1 = 1$ for $p = 4$
2. $c_1 = 1$: This leads to (32,0,0,32,0).

- $b_{12} = b_3 = 1$.

  Absence of $\widetilde{\mathrm{O3}}^-$ restricts the polynomial $\mathcal{C}(\{p_i\})$ to

$$\mathcal{C}(\{p_i\}) = c_{12}p_1p_2 + c_{13}p_1p_3 + c_{23}p_2p_3 + c_3p_3 \tag{5.2}$$

  with

$$c_{12} + c_{13} + c_{23} + c_3 = 0\,.$$

  This gives us 8 remaining possibilities to be checked. However, we still have some remaining symmetries that are generated by the transformations: $x_1 \to x_1 + x_2 + 1$, other coordinates invariant; $x_1 \leftrightarrow x_2$, other coordinates invariant; and, $x_1 \to x_1 + x_2$, $x_3 \to x_2 + x_3$. These leave $\mathcal{B}(\{p_i\})$ invariant but have a non-trivial effect on $\mathcal{C}(\{p_i\})$. Using these symmetries, one can reduce to 2 inequivalent options:

  1. $c_{12} = c_{13} = 1$: Leads to (32,16,0,16,0).
  2. $c_{12} = c_3 = 1$: Leads to (32,0,0,32,0).

- $b_{12} = b_{34} = b_5 = 1$.

  Again we require absence of $\widetilde{\mathrm{O3}}^-$ planes. A somewhat lengthy computation shows that this requirement restricts the polynomial $\mathcal{C}(\{p_i\})$ to

$$\mathcal{C}(\{p_i\}) = c_{12}p_1p_2 + c_{34}p_3p_4 + c_5p_5 \tag{5.3}$$

  with

$$c_{12} = c_{34} = c_5$$

  Hence the only new model comes from setting $c_{12} = c_{34} = c_5 = 1$, which gives (32,0,0,32,0).

The most tedious computations are the ones that will follow now. We found it convenient to invoke a simple computerprogram for parts of the analysis. Nevertheless, a number of computations was still done by hand. Equivalence or inequivalence of certain models can often be fairly easily checked, and allows us to avoid wasting computer time on configurations that have already been analyzed.

- $b_{12} = 1$. We use coordinate transformations in x_i with $i = 3, 4, 5, 6$ to bring the part of $\mathcal{C}(\{p_i\})$ with terms involving (only) these coordinates to standardform.

  We will start with the easiest case:

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1. $c_{34} = c_{56} = 1$.

   Redefining x_i $(i = 3,4,5,6)$ one can set c_{1i}, c_{2i} and c_i to zero. Using $x_1 \leftrightarrow x_2$, $x_1 \to x_1 + x_2 + 1$ also c_2 can be set to zero. The only possibly non-zero coefficients of $\mathcal{C}$ (apart from c_{34} and c_{56}) are then c_{12}, c_1, c. This gives in total 8 models, but a computation reveals that all have $r < 0$, and therefore they are not supersymmetric.

2. $c_{34} = 1$, $c_{56} = 0$.

   We may set coefficients c_{3i}, c_{4i}, c_3 and c_4 to zero by redefining x_3 and x_4. If $c_{15} = c_{16} = c_{25} = c_{26} = 0$, one sets $c_2 = 0$ by the transformations $x_1 \leftrightarrow x_2$ and $x_1 \to x_1 + x_2 + 1$. One can now also use $x_5 \leftrightarrow x_6$, and $x_5 \to x_5 + x_6$. to set c_6 to zero. If c_5 non-zero, one can transform to $c_1 = c = 0$. Summarizing, with $c_{34} = 1$, all $c_{ij} = 0$ except possibly c_{12}, we only need to consider non-zero values for c_{12}, c_1, c_5 and c. Computing r for these various possibilities one finds that $r \geq 0$ for:

   - $c = 0$: This is related by T-duality in the 3 and 4 directions to a model considered previously.
   - $c_{12} = 1$: Leads to the model (36,4,12,12,2).

   If $c_{15} = 1$, one may set c_{12}, c_{16} and c_1 to zero by redefining x_5. With $c_{16} = 1$ but $c_{15} = 0$ one uses $x_5 \leftrightarrow x_6$ .Thus in case of $c_{15} = c_{34} = 1$ we only need to consider non-zero c_2, c_5, c_6 and c. For these coefficients there is only one supersymmetric model:

   - $c_{15} = 1$: Leads to (32,8,16,8,0).

   With $c_{25} = 1$, but $c_{16} = c_{15} = 0$, one uses $x_1 \leftrightarrow x_2$. With $c_{15} = c_{25} = 1$, but $c_{16} = 0$ one uses $x_1 \to x_1 + x_2 + 1$. When $c_{25} = c_{16} = 1$ one can absorb any c_{1i} $(i \neq 6)$ and c_1 upon redefining x_6, and c_{2i} $(i \neq 5)$and c_2 upon redefining x_5. Hence for this subclass of theories one only has to consider non-zero-values for c_5, c_6 and c. All of these turn out to result in $r < 0$.

   With $c_{26} = 1$ but $c_{15} = c_{16} = c_{25} = 0$ one uses $x_1 \leftrightarrow x_2$ to reduce to a previously considered case. If $c_{26} = c_{15} = 1$, one uses $x_1 \leftrightarrow x_2$. If $c_{26} = c_{16} = 1$ use $x_5 \leftrightarrow x_6$. If $c_{26} = c_{15} = c_{16} = 1$ use $x_1 \leftrightarrow x_2$.

   In case $c_{26} = c_{25} = 1$, redefine x_5 to absorb c_{26}. Any remaining model with $b_{12} = c_{34} = 1$ can be transformed to one of the previously considered models, and we can pass on to the next class.

3. $c_{34} = c_{56} = 0$.

   As long as $c_{ij} = 0$, $\mathcal{C}(\{p_i\})$ takes the simpler form (3.13), we can use the same set of tricks as in the $p = 4$ case, and hence we only consider non-zero values for c_1 and c. This results in one supersymmetric model:

   - $c_1 = 1$: Leads to the model (32,0,16,16,0).

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If $c_{12} = 1$, we can set $c_2 = 0$ by using $x_1 \leftrightarrow x_2$ and $x_1 \to x_1 + x_2 + 1$. If $c_{12} = c_3 = 1$, one can absorb other c_i and c by redefining x_3. Considering the remaining possibilities gives the supersymmetric models:

- $c_{12} = 1$: Leads to (48,0,0,16,8).
- $c_{12} = c_1 = 1$: Leads to (32,16,16,0,0).

If at least one of the c_{1j} $(j \neq 2)$ is non-zero, one can use coordinate transformations in x_i $(i = 3, \ldots, 6)$ to set all c_{1j} except c_{13}, and c_1 to zero. If c_2 non-zero can set $c = 0$ by redefining $x_2 \to x_1 + x_2 + 1$. Possible values for c_i with $i > 3$ can all be absorbed in c_4 by using coordinate transformations in x_i with $i = 4, 5, 6$. Hence we capture all of these theories by setting $c_{13} = 1$, and considering all values for c_2, c_3, c_4 and c. This leads to the following set of supersymmetric models.

- $c_{13} = 1$: Related to a higher dimensional model by using T-duality in the 3-direction.
- $c_{13} = c_3 = 1$: Also related by T-duality in the 3-direction to a higher dimensional model.
- $c_{13} = c_3 = c_2 = 1$: Leads to (32,0,16,16,0).

We next consider $c_{23} = 1$, $c_{13} = 0$; in this case one uses $x_1 \leftrightarrow x_2$. If $c_{23} = c_{13} = 1$, use $x_1 \to x_1 + x_2 + 1$.

If $c_{23} = c_{14} = 1$, one can redefine x_3 and x_4 to absorb all other c_{1j}, c_{2j}, c_1 and c_2. If either one of c_3 or c_4 is zero, they can both be set to zero in the following way: If $c_3 = 0$ and $c_4 = 1$, use $x_1 \to x_1 + x_2 + 1, x_3 \to x_3 + x_4$; If $c_3 = 1$ and $c_4 = 0$, use $x_2 \to x_1 + x_2 + 1, x_4 \to x_3 + x_4$. Hence in this case we only need to check for values of $c_3 = c_4$ and c. This results in a single supersymmetric model

- $c_{23} = c_{14} = 1$: Leads to (32,8,16,8,0) .

If $c_{23} = 1$ and $c_{15} = 1$ and/or $c_{16} = 1$ one uses coordinate transformations in x_4, x_5, x_6 to set $c_{14} = 1$ and proceeds as previously.

If $c_{24} = 1$ and $c_{23} = 0$ one uses $x_3 \leftrightarrow x_4$. If $c_{24} = c_{23} = 1$ then redefine x_3 to absorb all other c_{2j} and c_2, and hence set $c_{24} = 0$. Finally, if c_{25} or c_{26} is non-zero, one again permutes x_4, x_5, x_6, such that c_{24} is set to one, and proceeds as before.

This finishes the computation of configurations with $b_{12} = 1$

- $b_{12} = b_{34} = 1$.

We would like to bring the part with quadratic terms of the expression for $\mathcal{C}(\{p_i\})$ to standardform, but our freedom is limited to the coordinates x_5 and x_6. Therefore the "standardform" simply amounts to stating the value of c_{56},

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all other symmetries have to come from the transformations that leave $\mathcal{B}(\{p_i\})$ invariant.

1. $c_{56} = 1$.

   In principle one could go through the whole tedious procedure of the fixing of symmetries. However, running a computer program computing all possible configurations with $b_{12} = b_{34} = c_{56} = 1$ (without actually fixing any other symmetries) shows that none of these theories is supersymmetric. Therefore our efforts would be in vain anyway, and we simply pass on to $c_{56} = 0$.

2. $c_{56} = 0$.

   At first we set $c_{i5}, c_{j6} = 0$.

   Symmetries of the B-field configuration are respected by a group of symmetries (see $p = 4$), generated by

$$(x_1, x_2, x_3, x_4) \to (x_2, x_1, x_3, x_4) \tag{5.4}$$

$$(x_1, x_2, x_3, x_4) \to (x_1 + x_2 + 1, x_2, x_3, x_4) \tag{5.5}$$

$$(x_1, x_2, x_3, x_4) \to (x_3, x_4, x_1, x_2) \tag{5.6}$$

$$(x_1, x_2, x_3, x_4) \to (x_1 + x_3, x_2, x_3, x_2 + x_4)\,. \tag{5.7}$$

   Of course there are also symmetries involving x_5 and x_6, but because $\mathcal{B}(\{p_i\})$ does not involve these coordinates, we do not mention them at this point.

   The set of coefficients $c_{12}, c_{13}, c_{14}, c_{23}, c_{24}$ and c_{23} can a priori take 64 values, but under the above symmetries many of these are related. We decompose the orbits of the symmetries on the space of possible values of the set of above c_{ij}'s (there turn out to be 6 of these) and pick a representative from each orbit. Then we run a computer calculation on each representative, checking on supersymmetric configurations if combinations of c_i's and c are turned on. This immediately leads to the following results:

   - $c_{ij} = 0$: If not $c_i = c = 0$ then supersymmetry is broken. The remaining configuration has $\mathcal{C}(\{p_i\}) = 0$ and can be found in section 4.1.
   - $c_{12} = 1$: Only $c_i = c = 0$ is a supersymmetric configuration; this leads to (36,12,4,12,2).
   - $c_{13} = 1$: Supersymmetric configurations are $c_i = 0$, $c_1 = 1$ and $c_3 = 1$. These are equivalent by symmetries: $x_1 \to x_1 + x_3, x_4 \to x_2 + x_4$, takes $c_{13} = 1, c_i = 0$ to $c_{13} = c_3 = 1$, while $x_3 \to x_1 + x_3, x_2 \to x_2 + x_4$, takes $c_{13} = 1, c_i = 0$ to $c_{13} = c_1 = 1$. Therefore they are really only one model, with characteristic (32,16,8,8,0).

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- $c_{13} = c_{24} = 1$: Supersymmetric configurations are $c_i = 0$, $c_2 = c_3 = 1$ and $c_1 = c_4 = 1$. These are also related to each other by the symmetries $x_1 \to x_1+x_3, x_4 \to x_2+x_4$, and $x_3 \to x_1+x_3, x_2 \to x_2+x_4$, and hence all lead to the same model, (32,8,8,16,0).
- $c_{12} = c_{13} = c_{24} = 1$: None of the theories in this orbit is supersymmetric.
- $c_{12} = c_{34} = 1$: Only $c_1 = c = 0$ is supersymmetric. This leads to the model (40,0,0,24,4).

Next consider $c_{15} = 1$. We can now absorb other c_{1i} and c_1 by redefining x_5. Combining this with the orbit structure one is left with:

- $c_{15} = 1$: The only supersymmmetric model has $c_i = c = 0$, but is related by T-duality to a higher dimensional model.
- $c_{15} = c_{24} = 1$: All values for c_i and c give tadpoles.
- $c_{15} = c_{34} = 1$: The only supersymmetric model is $c_i = c = 0$, leading to (32,8,8,16,0).

If $c_{16} = 1$ we use $x_5 \leftrightarrow x_6$. If $c_{15} = c_{16} = 1$ one absorbs c_{16} by redefining x_5.

Next we turn to $c_{25} = 1$. Now we absorb all other c_{2i} and c_2 in a redefinition of x_5

- $c_{25} = 1$: use $x_1 \leftrightarrow x_2$.
- $c_{13} = c_{25} = 1$: use $x_1 \leftrightarrow x_2, x_3 \leftrightarrow x_4$.
- $c_{34} = c_{25} = 1$: use $x_1 \leftrightarrow x_2$.

If $c_{15} = c_{25} = 1$, one first absorbs the other c_{1i} and c_{2i} in a redefinition of x_5. Then use $x_1 \to x_1 + x_2 + 1$ to eliminate c_{25}.

If $c_{25} = c_{16} = 1$, one first absorbs the other c_{1i} and c_{2j} in redefinitions of x_5 and x_6. The only two-index coefficient in $\mathcal{C}(\{p_i\})$ that is not fixed yet is c_{34}, but a check reveals that both $c_{34} = 0$ and $c_{34} = 1$ always result in supersymmetry breaking configurations. When $c_{25} = c_{16} = c_{15} = 1$ one again absorbs c_{1i} and c_{2i} (with $i \neq 5$) in a redefinition of x_5, and proceeds as previously. With $c_{26} = 1, c_{25} = 0$, use $x_5 \leftrightarrow x_6$. If $c_{26} = c_{25} = 1$: Redefine x_5 to absorb c_{26}.

Next consider $c_{35} = 1$. Absorb all other c_{3i} in a redefinition of x_5. The following options remain:

- $c_{35} = 1$: Use $x_1 \leftrightarrow x_3$, $x_2 \to x_4$.
- $c_{12} = c_{35} = 1$: Use $x_1 \leftrightarrow x_3, x_2 \leftrightarrow x_4$.
- $c_{24} = c_{35} = 1$: Use $x_1 \leftrightarrow x_3, x_2 \leftrightarrow x_4$.
- $c_{12} = c_{24} = c_{35} = 1$: These all break supersymmetry.

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If $c_{35} = c_{15} = 1$, one absorbs all other c_{3i} and c_{1i} in a redefinition of x_5. The only coefficient left to consider is $c_{24} = 0, 1$, but now one can always use $x_1 \to x_1 + x_3, x_4 \to x_2 + x_4$ to relate this to a previously considered model. With $c_{35} = c_{16} = 1$. Again one absorbs all other c_{1i} and c_{3i} in redefinitions of x_5 and x_6, and one is left with only $c_{24} = 0, 1$. All possible models with these coefficients break supersymmetry.

With $c_{35} = c_{25} = 1$ absorb all other c_{2i} and c_{3i} in a redefinition of x_5. Then use $x_2 \leftrightarrow x_2 + x_3, x_4 \to x_1 + x_4$. The cases $c_{35} = c_{25} = 1$ with $c_{15}, c_{16} = 0, 1$ are treated similarly. With $c_{35} = c_{26} = 1$, first transform away c_{2i} and c_{3i}, then use $x_1 \leftrightarrow x_2$. With $c_{35} = c_{26} = c_{25} = 1$, transform away c_{2i}, c_{3i} and c_{26}. With $c_{36} = 1$ use $x_5 \leftrightarrow x_6$. With $c_{35} = c_{36} = 1$, absorb c_{36} in a redefinition of x_5.

With $c_{45} = 1$ absorb all other c_{4i} in a redefinition of x_5. After this one is left with possible non-zero values for c_{12} and c_{13}. By using $x_1 \leftrightarrow x_4, x_2 \leftrightarrow x_3$ this can always be reduced to a previously considered model. With $c_{45} = c_{15} = 1$ one redefines x_5 to absorb other c_{1i} and c_{4i}. Then use $x_1 \to x_1 + x_4, x_3 \to x_2 + x_3$. With $c_{45} = c_{16} = 1$, absorb other c_{1i} and c_{4i} in redefinitions of x_5 and x_6. Then use $x_1 \leftrightarrow x_2, x_3 \leftrightarrow x_4$. With $c_{45} = c_{25} = 1$, first absorb c_{2i} and c_{4i} in a redefinition of x_5. Then use $x_2 \to x_2 + x_4, x_3 \to x_1 + x_3$. With $c_{45} = c_{25} = 1$ and c_{15} and/or c_{16} is 1, absorb all other c_{1i}, c_{2i} and c_{4i} with coordinate definitions of x_5 and x_6, and use again $x_2 \to x_2 + x_4, x_3 \to x_1 + x_3$. With $c_{45} = c_{26} = 1$, use $x_1 \leftrightarrow x_2$, $x_3 \leftrightarrow x_4$.

With $c_{45} = c_{35} = 1$, absorb all other c_{3i} and c_{4i} in a redefinition of x_5, and then use $x_3 \to x_3 + x_4 + 1$. With $c_{45} = c_{36} = 1$, absorb all other c_{3i} and c_{4i} and use $x_1 \leftrightarrow x_3, x_2 \leftrightarrow x_4$. The same for $c_{45} = c_{36} = c_{35} = 1$.

Finally, for $c_{46} = 1$ and $c_{45} = 0$, one uses $x_5 \leftrightarrow x_6$, and for $c_{46} = c_{45} = 1$ one can use $x_5 \to x_5 + x_6$.

We have explicitly or implicitly considered all models with $b_{12} = b_{34} = 1$, and we pass on to the last category of theories.

- $b_{12} = b_{34} = b_{56} = 1$.

There are only relatively few, to be precise 4 D3 branes to be possibly distributed over the O3 planes. One possibility is requiring absence of $\widetilde{\mathrm{O3}}^-$ planes, which automatically implies $c_{12} = c_{34} = c_{56}$, all others zero. Therefore, the only non-trivial solution to this constraint is

  - $c_{12} = c_{34} = c_{56} = 1$ leading to (36,0,0,28,2).

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Running a check with the computer on other configurations with $b_{12} = b_{34} = b_{56} = 1$ immediately reveals that there are no other supersymmetric solutions, than the above one and the one with $\mathcal{C}(\{p_i\}) = 0$.

In this subsection we have identified in total 17 new models, that were collected in table 4.

6. S-duality in 4 dimensions

All our orientifold theories on $T^6/\mathbb{Z}_2$ result in low energy effective 4 dimensional $\mathcal{N} = 4$ supersymmetric Yang-Mills theories. As is well known, these theories exhibit an $\mathrm{SL}(2,\mathbb{Z})$ S-duality symmetry. This provides an interesting check on our results and methods.

First of all we note that the S-duality symmetry was already manifest in our formalism. In 4 dimensions the RR charges are described by (3.14), which is isomorphic to the formula for the NS charges (3.8). The group $\mathrm{SL}(2,\mathbb{Z})$ is generated by

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}. \tag{6.1}$$

Because the formula's (3.8) and (3.14) are over the field $\mathbb{Z}_2$, the manifest S-duality group is not $\mathrm{SL}(2,\mathbb{Z})$, but its reduction modulo 2, $\mathrm{SL}(2,\mathbb{Z}_2)$. This group is generated by matrices S and T, reduced modulo 2. The effect of the matrix S is the interchange of the formula's (3.8) and (3.14). The transformation T leaves (3.14) invariant, but replaces (3.8) by

$$(\mathcal{B}+\mathcal{C})(\{p_i\}) \equiv \int_{M_p} (B+C) = \sum_{i<j} (b_{ij}+c_{ij}) p_i p_j + \sum_i (b_i + c_i) p_i + (b+c)\,. \tag{6.2}$$

Recall that $\mathcal{B}(\{p_i\})$ gave 1 on $\mathrm{O3}^+$ and $\widetilde{\mathrm{O3}}^+$, and 0 on $\mathrm{O3}^-$ and $\widetilde{\mathrm{O3}}^-$, whereas $\mathcal{C}(\{p_i\})$ resulted in 1 on $\widetilde{\mathrm{O3}}^+$ and $\widetilde{\mathrm{O3}}^-$, and 0 on $\mathrm{O3}^+$ and $\mathrm{O3}^-$. Hence formula (6.2) gives the value 1 on $\mathrm{O3}^+$ and $\widetilde{\mathrm{O3}}^-$, and 0 on $\mathrm{O3}^-$ and $\widetilde{\mathrm{O3}}^+$.

From the action of S and T on the formula's describing the charges we see that S interchanges $\mathrm{O3}^+$ with $\widetilde{\mathrm{O3}}^-$, and therefore n_+ with $\tilde{n}_-$, while T interchanges $\mathrm{O3}^+$ with $\widetilde{\mathrm{O3}}^+$, and hence $n_+ \leftrightarrow \tilde{n}_+$. These two transformations generate the whole permutation group $\mathcal{S}_3 \cong \mathrm{SL}(2,\mathbb{Z}_2)$, acting on the 3 numbers in the triple $(\tilde{n}_-, n_+, \tilde{n}_+)$.

One can now distinguish essentially 3 possibilities. First, if $\tilde{n}_- = n_+ = \tilde{n}_+$, then the configuration forms a singlet under $\mathrm{SL}(2,\mathbb{Z}_2)$. There are actually 2 models in our tables with this property: the trivial compactification, with (64,0,0,0,64), and a much more interesting model, with (40,8,8,8,4).

Instead of the 3 numbers $\tilde{n}_-$, n_+, $\tilde{n}_+$ being all equal, they could also be all different. This possibility is however not realized at all, as can be seen by inspection

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of our tables. We don't know whether this is just a numerical coincidence, or there is a deeper level of understanding of this fact possible.

All configurations in our tables that are not singlets under $\mathrm{SL}(2,\mathbb{Z}_2)$ have 2 out of the 3 numbers $\tilde{n}_-$, n_+, $\tilde{n}_+$ equal. In this case, some transformations in $\mathrm{SL}(2,\mathbb{Z}_2)$ have a trivial effect on the triple $(\tilde{n}_-, n_+, \tilde{n}_+)$. Only a $\mathbb{Z}_3$ subgroup is manifest, and the orbits of the group consist of 3 elements.

It is a nice consistency check on our results to divide the models in our tables in orbits under $\mathrm{SL}(2,\mathbb{Z}_2)$. Closure of all orbits gives us added confidence that our classification is indeed complete. In most cases this is a fairly simple exercise, as often the set of numbers $(\tilde{n}_-, n_+, \tilde{n}_+)$ specifies the model completely (in 4 dimensions $n_- = 64 - \tilde{n}_- - n_+ - \tilde{n}_+$, and r follows from (3.17)).

There exist however models with equal $(n_-, n_+, \tilde{n}_-, \tilde{n}_+, r)$ that are nevertheless not equivalent. Earlier examples of models with this property involved Op^- and Op^+ planes only [2, 3, 16] (see our table 1), but we now see that this phenomenon also occurs in the mixed cases. We now also need the information provided by the coefficients of $\mathcal{B}(\{p_i\})$ and $\mathcal{C}(\{p_i\})$ as these specify the geometry of all planes.

In this case we always have $r = 0$ (we do not know whether there is a deeper reason that such models should have $r = 0$, it follows from inspection of the tables). For some configurations whose $\mathrm{SL}(2,\mathbb{Z}_2)$ orbit cannot be determined by the set of numbers $(n_-, n_+, \tilde{n}_-, \tilde{n}_+, r)$, we have attached an index to the number $r = 0$ in tables 3 and 4. Theories with the same index belong to the same $\mathrm{SL}(2,\mathbb{Z}_2)$ orbit. The theories in table 4 with $r = 0$, but not labelled by an index are S-dual to theories in tables 1 and 2. The reader should have no difficulty identifying these models and their duals.

Rather interestingly, our whole formalism is $\mathrm{SL}(2,\mathbb{Z})$ dual, regardless whether in the end one finds a supersymmetric theory or not, and therefore also assigns dual theories to non-supersymmetric theories. It would be interesting to see to what extent physics in these non-supersymmetric "dual" theories is related to each other, the more because after the anti-brane annihilation process that turns $\widetilde{\mathrm{O}p}^-$ into a bound state of Op^- with an anti Dp-brane, the manifest duality between various descriptions will have dissappeared.

7. Conclusions

By computing the holonomies of gerbes defined by the discrete charges of the orientifold planes, the problem of classification of orientifold planes on $T^k/\mathbb{Z}_2$ can be reduced to rather simple polynomial equations over the field $\mathbb{Z}_2$. Moreover, these polynomials represent a very compact way of storing information about the orientifold model, as many relevant quantities can be deduced from them. The techniques we used have their roots in [3], where they were applied to K3's of the form T^4/G (G a finite group), but are useful in the present problem as well. In principle they

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d	$r=16$	$r=8$	$r=4$	$r=2$	$r=0$	total
9	1	0	0	0	0	1
8	1	0	0	0	1	2
7	1	1	0	0	1	3
6	1	1	0	0	2	4
5	1	2	1	1	2	7
4	1	2	2	1	7	13
3	1	3	4	6	21	35

Table 5: Number of orientifold theories in d dimensions for all values of r

may be relevant to any situation involving orientifold and/or orbifold fixed point sets with more than one connected component, in the presence of discrete torsion and similar degrees of freedom.

Preservation of supersymmetry reduces to a simple inequality. With these powerful techniques the classification of supersymmetric orientifolds was carried out for all $k \leq 6$. The results can be found in the tables 1, 2, 3 and 4.

All our theories are dual to toroidal compactifications of either type I theory, or the type IIB theory on $S^1/\delta\Omega$, with suitable holonomies on the torus. This has not played a large role in our analysis in this paper. Instead we refer the reader to [3, 14, 16, 18, 22] for various ideas and results.

For the benefit of the reader, we collect some information from our 4 tables in another table summarizing our results.

Table 5 lists the number of maximally supersymmetric orientifold models in various dimensions, where d is the number of spatial dimensions. It should be stressed that this is not a classification of irreducible components in the string moduli space. On the one hand, there are many components in the string moduli space that do not have an orientifold description. On the other hand, it is known that a single irreducible component in the string moduli space can give rise to multiple orientifold descriptions [3].

One of the things that immediately draws attention is the explosive growth of number of theories for $d < 6$. This can be viewed as the analogue of the existence of triples, and other non-standard compactifications in gauge theory, where for sufficiently high compact dimension there are many irreducible components in the moduli space of flat connections [4, 12, 13]. Another typical fact is that theories with $r = 4$ and $r = 2$ appear simultaneously, at $d = 5$, but that then subsequently there are no new values for r encountered in the rest of the table. Furthermore, except for $r = 0$, the numbers r are even powers of 2. Comparing with for example [4, 14], one sees that this does not follow from the possible bundles on T^k (as there exist flat $\mathrm{Spin}(32)/\mathbb{Z}_2$ bundles with rank 12 and rank 5, for example), and not even from supersymmetry (the previously mentioned bundles give perfectly valid vacua for su-

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persymmetric gauge theories on tori), but from constraints that are intrinsic to string theory [3]. It would be nice to understand the numerology better[6].

We also discussed the implications of S-duality of 4-dimensional $\mathcal{N} = 4$ supersymmetric gauge theories. Among the orientifold configurations we found, there are two singlets under the $\mathrm{SL}(2, \mathbb{Z})$ duality group, one with $r = 16$ (64,0,0,0,16) and one with $r = 4$ (40,8,8,8,4). Especially the latter one may be an interesting theory to study, the behavior under S-duality suggests a highly symmetric spectrum, and with $r = 4$ gauge groups of relatively large rank are possible. Note that it is even possible to occupy at least one of each kind of Op-plane with a Dp-brane pair. It may also be interesting to compute the lattice for the heterotic version of this theory, along the lines of [3]. The remaining theories organize in orbits of each 3 elements, which is reflected in table 5, because if one omits the singlets, all the numbers in the last line of the table are divisible by 3.

Although discussed in less detail, it should also be clear that the present techniques provide a rich source of examples of non-supersymmetric theories, similar to the ones analyzed in [19, 21].

To take this classification to still lower dimension, we need new insights concerning the rewriting of (3.15) to some kind of standardform. An estimate on the needed time to perform a computer analysis without any new information, does not give much hope that continuing this classification to $p = 2$ by brute force can be done in the near future.

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